



24

Synthetic division *10 points*

Introduction

What a name! The synthetic division refers to the method to divide a polynomial by the binomial $(x - c)$ where c is a constant. Consider the case of dividing

$$(-3x^3 + 5x - 2)/(x - 5)$$

Do not be afraid since Ruffini's rule will help you to do so. Let's see how it works with the previous example. First, it begins by drawing a couple of crossed lines and put the c value at left.

$$\begin{array}{r|l}
 5 & \\
 \hline
 &
 \end{array}$$

Next step is to write the coefficients of the polynomial ordered from highest to lowest degree at the top. If some degree is missing, put it as a zero in its corresponding place. In this case the coefficients are -3, for x^3 , 5 for x and -2 as an independent term.

$$\begin{array}{r|rrrr}
 5 & -3 & 0 & 5 & -2 \\
 \hline
 & & & &
 \end{array}$$

Now copy the coefficient of highest degree, which is -3, at the top just under horizontal line.

$$\begin{array}{r|rrrr}
 5 & -3 & 0 & 5 & -2 \\
 \hline
 & -3 & & &
 \end{array}$$

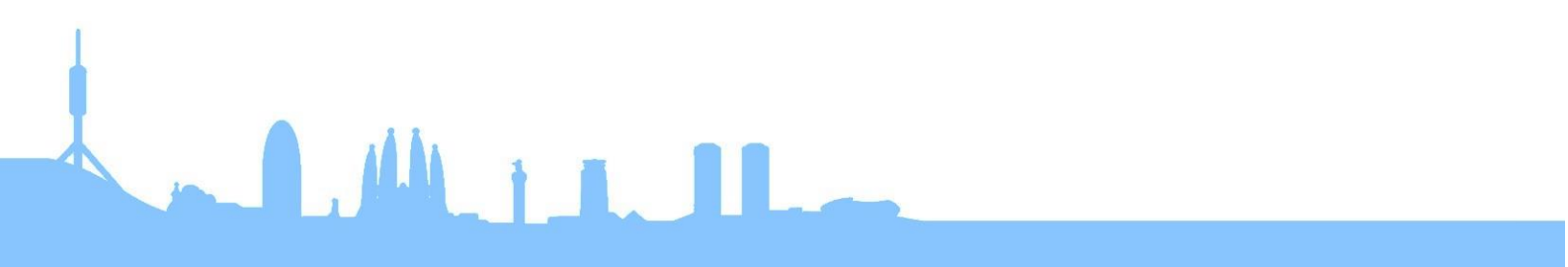
Multiply this number by the value of c , which is 5, and the result is put next above the horizontal line.

$$\begin{array}{r|rrrr}
 5 & -3 & 0 & 5 & -2 \\
 & & -15 & & \\
 \hline
 & -3 & & &
 \end{array}$$

Then add the values in the second column, write the result under the horizontal line and repeat the multiplication with the value of c .

$$\begin{array}{r|rrrr}
 5 & -3 & 0 & 5 & -2 \\
 & & -15 & -75 & \\
 \hline
 & -3 & -15 & &
 \end{array}$$

Again, it is time to add the numbers in the column and put the result down the horizontal line.





$$\begin{array}{r|rrrr}
 & -3 & 0 & 5 & -2 \\
 5 & & -15 & -75 & \\
 \hline
 & -3 & -15 & -70 &
 \end{array}$$

Repeat these steps until reaching the last column.

$$\begin{array}{r|rrrrr}
 & -3 & 0 & 5 & -2 \\
 5 & & -15 & -75 & -350 \\
 \hline
 & -3 & -15 & -70 & -352
 \end{array}$$

Last number at the right, that is -352, is the remainder of the division. And the polynomial quotient of the division is built from the coefficient numbers that are previous to the remainder from left to right providing as a result $-3x^2 - 15x - 70$

Now that you have refreshed how the Ruffini's rule work, can you write a program to perform a synthetic division?

Important note: The expected length of the horizontal line is 5 dashed characters per each number plus an extra dash aligned with the vertical line.

Input

Two lines form the input. First line contains the coefficients of the dividend where a zero represents any missing terms. Second line have a single number representing the c constant of the binomial divisor.

Output

The final table after applying Ruffini's rule. Please note that per each number a fixed size of 5 positions is defined in order to have the number properly printed in columns.

Example

Input

```
-3 0 5 -2
5
```

Output

$$\begin{array}{r|rrrr}
 & -3 & 0 & 5 & -2 \\
 5 & & -15 & -75 & -350 \\
 \hline
 & -3 & -15 & -70 & -352
 \end{array}$$

