While iterations X38759_en

You have to program several functions. Do not use the math module.

1. Write an integer function *int_root*(n) that given a natural number n returns $\lfloor \sqrt{n} \rfloor$.
2. Write a function *int_log*(a, b) that given natural numbers a greater than one and b greater than zero returns natural k such that $a^k \leq b < a^{k+1}$.
3. Write a function *gcd_lcm*(a, b) that given natural numbers a and b such that $a \neq 0$ or $b \neq 0$ returns the greatest common divisor and the least common multiple. Your code has to implement the Euclid's algorithm.
4. Write a boolean function *is_prime*(n) that given a natural number n returns `True` if and only if n is prime.
5. In order to play table games at the casino you need some tokens. Red tokens cost 7 euros and yellow tokens cost 4. Write a function *buy_tokens*(n) that given a number n of euros such that $n \geq 20$, it returns the equivalence in tokens. When several equivalences are possible the function returns the one minimizing the total number of tokens.
6. Write a function *inv_factorial*(n) that given an integer $n > 1$ it returns the number m such that $(m-1)! < n \leq m!$.

Scoring

The first function counts 15 points. Other ones count 17 point each one.

Sample session

```
>>> int_root(19)
4
>>> int_log(3, 20)
2
>>> gcd_lcm(12,18)
(6, 36)
>>> is_prime(51)
False
>>> buy_tokens(50)
(6, 2)
>>> inv_factorial(50)
5
```

Problem information

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