31 Latin squares

17 points

Introduction

A latin square is an n×n array filled with n different symbols, each occurring exactly once in each row and exactly once in each column. Here is an example:

A	B	C
C	A	B
B	C	A

A common representation of a Latin square is as an array of triple (r,c,s), where r is the row, c is the column, and s is the symbol. For example, the representation of the following Latin square is:

A	B	C
B	C	A
C	A	B

{ (1,1,A),(1,2,B),(1,3,C),(2,1,B),(2,2,C),(2,3,A),(3,1,C),(3,2,A),(3,3,B) } where for example the triple (2,3,A) means that the cell in row 2 and column 3 contains the symbol A.

Two Latin squares of the same order n called L_1 and L_2 are orthogonal if, for each ordered pair of symbols (k,k') there is one and only one position (i,j) where $L_1(i,j) = k$ and $L_2(i,j)=k'$

For example, the following two Latin squares are orthogonal as each of the pairs (A,A), (A,B),…,(D,D) just appears in one of the 16 positions.

A	B	C	D
B	A	D	C
C	D	A	B
D	C	B	A

A	B	C	D
C	D	A	B
D	C	B	A
B	A	D	C

Write a program that reads in the first line an array representation of a Latin square of arbitrary order n and that for the rest of the lines gets array representations of *n*×*n* matrices with the same set of symbols of the first line Latin squares. The program must output those arrays being orthogonal Latin squares to the first one.

Assumptions:

- Symbols are alphanumeric characters and format can have spare spaces.
- Indexes start at 1.
- Lines can be empty (just carriage return), in that case, the row will be considered as a non-Latin square.
- Any of the rows can have a format error, in that case, the row will be considered as a non-Latin square.
- If the first line is not a Latin square none could be orthogonal to it.
- The input contains at least two lines.

Input

The first line is an array representation of a Latin square of order n, followed by other lines that represent nxn matrices with the same set of symbols.

```
{ (1,1,A),(1,2,B),(1,3,C),(2,1,C),(2,2,A),(2,3,B),(3,1,B),(3,2,C),(3,3,A) }
```

```
{ (1,1,A),(1,2,B),(1,3,C),(2,1,B),(2,2,C),(2,3,A),(3,1,C),(3,2,A),(3,3,B) }
{ (1,1,B),(1,2,C),(1,3,A),(2,1,C),(2,2,A),(2,3,B),(3,1,A),(3,2,B),(3,3,C) }
{ (1,1,A),(1,2,A),(1,3,A),(2,1,B),(2,2,C),(2,3,A),(3,1,C),(3,2,A),(3,3,B) }
{ (1,1,B),(1,2,A),(1,3,C),(2,1,C),(2,2,B),(2,3,A),(3,1,A),(3,2,C),(3,3,B) }
```

Output

The output is the list of arrays being orthogonal Latin squares for the provided in the first line of the input. In this case, those in lines 2 and 3, since 4 is not a Latin square and 5 is not orthogonal.

```
{ (1,1,A),(1,2,B),(1,3,C),(2,1,B),(2,2,C),(2,3,A),(3,1,C),(3,2,A),(3,3,B) }
{ (1,1,B),(1,2,C),(1,3,A),(2,1,C),(2,2,A),(2,3,B),(3,1,A),(3,2,B),(3,3,C) }
```