Symmetric polynomials **P75097_en**

Vint-i-tresè Concurs de Programació de la UPC - Final (2025-09-17)

A polynomial p in three variables a, b and c is *symmetric* if and only if $p(a,b,c) = p(a,c,b) = p(b,a,c) = \dots$ for the six permutations of the variables.

For example, $a+b+c$, $ab+bc+ac$, $3a^2b^2c^2$ and $7abc+a^2bc+ab^2c+abc^2$ are symmetric polynomials, while $ab+ac$ and $a^2bc-ab^2c+abc^2$ are not.

We introduce the notation `[a^i b^j c^k]` with $i \geq j \geq k$ to denote the symmetric polynomial that results from adding all the monomials of the form $a^ib^jc^k$ for any permutation of a, b and c, where all the resulting monomials appear with coefficient 1. For example, `[a]` $= a+b+c$, `[ab]` $= ab+bc+ca$, and `[a^2bc]` $= a^2bc+ab^2c+abc^2$. (Note the special cases for the notation when the exponent of a variable is zero or one.)

Symmetric polynomials that do not have any variables of degree larger than one, that is, `[a]`, `[ab]` and `[abc]`, are called *elementary* symmetric polynomials. The fundamental theorem of symmetric polynomials, already known to Newton, states that *any* symmetric polynomial can be expressed as the sum and product of elementary symmetric polynomials.

Here, we don't ask you to find these expressions. Instead, we ask you a much simpler task: calculate the product between a symmetric polynomial `[a^i b^j c^k]` and an elementary symmetric polynomial. (If you do this, you are not far away from establishing a recurrence relation and explicitly finding the expressions from the fundamental theorem.)

Input

Input consists of several cases, each with the product of a symmetric polynomial and an elementary symmetric polynomial. Assume $i \geq 1$ and $0 \leq k \leq j \leq i \leq 1000$.

Output

For every product, print its result. Make sure that the terms are in lexicographical order, that is, first the term with the largest i, and in case of a tie, first the term with the largest j.

Sample input

```
[a]*[a]
[ab]*[a]
[a^2b]*[ab]
[a^3b]*[ab]
[a^3b]*[abc]
[a^1000b^700c^42]*[ab]
```

Sample output

```
[a^2] + 2[ab]
[a^2b] + 3[abc]
[a^3b^2] + 2[a^3bc] + 2[a^2b^2c]
[a^4b^2] + 2[a^4bc] + [a^3b^2c]
[a^4b^2c]
[a^1001b^701c^42] + [a^1001b^700c^43] + [a^1000b^701c^43]
```

Problem information

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Generation : 2025-09-15 20:08:26