

**Monic irreducible polynomials**

**P64196\_en**

Setè Concurs de Programacio de la UPC - Final (2009-09-16)

Here, we consider polynomials in  $\mathbb{F}_p[x]$ , that is, polynomials on  $x$  whose coefficients are elements of  $\mathbb{F}_p = \{0, 1, 2, \dots, p - 1\}$ , where  $p$  is a prime number.

A polynomial is *monic* if the coefficient of its term with largest degree is 1. A polynomial is *irreducible* if it cannot be written as the product of two polynomials of smaller degree. Your task is to count the number of monic, irreducible polynomials of  $\mathbb{F}_p[x]$  of a given degree  $d$ .

Too difficult? Do not despair! The problem is not so hard, once you know that, in  $\mathbb{F}_p[x]$ , every monic polynomial can be written in a unique way as a factor of monic, irreducible polynomials. For instance, in  $\mathbb{F}_2[x]$  there are 4 monic polynomials of degree 2 (in  $\mathbb{F}_2[x]$ , all polynomials are monic), but only one of them is irreducible:

$$x^2 = x \cdot x \quad x^2 + 1 = (x + 1) \cdot (x + 1) \quad x^2 + x = x \cdot (x + 1) \quad x^2 + x + 1 = ???$$

In  $\mathbb{F}_2[x]$ , there are 8 monic polynomials of degree 3, but only two of them are irreducible:

$$\begin{array}{ll} x^3 = x \cdot x \cdot x & x^3 + x^2 = x \cdot x \cdot (x + 1) \\ x^3 + 1 = (x + 1) \cdot (x^2 + x + 1) & x^3 + x^2 + 1 = ??? \\ x^3 + x = x \cdot (x + 1) \cdot (x + 1) & x^3 + x^2 + x = x \cdot (x^2 + x + 1) \\ x^3 + x + 1 = ??? & x^3 + x^2 + x + 1 = (x + 1) \cdot (x + 1) \cdot (x + 1) \end{array}$$

**Input**

Input consists of several cases, each with a prime number  $2 \leq p \leq 30$  and an integer number  $2 \leq d \leq 30$ . Additionally, we have  $p^d < 10^9$ .

**Output**

For every case, print the number of monic, irreducible polynomials in  $\mathbb{F}_p[x]$  of degree  $d$ .

**Sample input**

```
2 2
2 3
2 4
2 30
3 2
3 3
3 4
3 19
29 6
```

**Sample output**

```
1
2
3
35790267
3
8
18
61171656
99133020
```

**Problem information**

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