Monic irreducible polynomials
Setè Concurs de Programacio de la UPC - Final (2009-09-16)

Here, we consider polynomials in \( \mathbb{F}_p[x] \), that is, polynomials on \( x \) whose coefficients are elements of \( \mathbb{F}_p = \{0, 1, 2, \ldots, p - 1\} \), where \( p \) is a prime number.

A polynomial is **monic** if the coefficient of its term with largest degree is 1. A polynomial is **irreducible** if it cannot be written as the product of two polynomials of smaller degree. Your task is to count the number of monic, irreducible polynomials of \( \mathbb{F}_p[x] \) of a given degree \( d \).

Too difficult? Do not despair! The problem is not so hard, once you know that, in \( \mathbb{F}_p[x] \), every monic polynomial can be written in a unique way as a factor of monic, irreducible polynomials. For instance, in \( \mathbb{F}_2[x] \) there are 4 monic polynomials of degree 2 (in \( \mathbb{F}_2[x] \), all polynomials are monic), but only one of them is irreducible:

\[
x^2 = x \cdot x \quad x^2 + 1 = (x + 1) \cdot (x + 1) \quad x^2 + x = x \cdot (x + 1) \quad x^2 + x + 1 = ???
\]

In \( \mathbb{F}_2[x] \), there are 8 monic polynomials of degree 3, but only two of them are irreducible:

\[
x^3 = x \cdot x \cdot x \quad x^3 + x^2 = x \cdot x \cdot (x + 1) \\
x^3 + 1 = (x + 1) \cdot (x^2 + x + 1) \quad x^3 + x^2 + 1 = ??? \\
x^3 + x = x \cdot (x + 1) \cdot (x + 1) \quad x^3 + x^2 + x = x \cdot (x^2 + x + 1) \\
x^3 + x + 1 = ??? \quad x^3 + x^2 + x + 1 = (x + 1) \cdot (x + 1) \cdot (x + 1)
\]

**Input**

Input consists of several cases, each with a prime number \( 2 \leq p \leq 30 \) and an integer number \( 2 \leq d \leq 30 \). Additionally, we have \( p^d < 10^9 \).

**Output**

For every case, print the number of monic, irreducible polynomials in \( \mathbb{F}_p[x] \) of degree \( d \).

**Sample input**

<table>
<thead>
<tr>
<th>2 2</th>
<th>3 3</th>
<th>4 4</th>
<th>2 30</th>
<th>2 2</th>
<th>3 2</th>
<th>3 3</th>
<th>3 4</th>
<th>3 19</th>
<th>29 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

**Sample output**

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>35790267</th>
<th>3</th>
<th>8</th>
<th>18</th>
<th>61171656</th>
<th>99133020</th>
</tr>
</thead>
<tbody>
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**Problem information**

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