Monic irreducible polynomials — P64196_en

Setè Concurs de Programacio de la UPC - Final (2009-09-16)

Here, we consider polynomials in $\mathbb{F}_p[x]$, that is, polynomials on x whose coefficients are elements of $\mathbb{F}_p = \{0, 1, 2, \ldots, p-1\}$, where p is a prime number.

A polynomial is *monic* if the coefficient of its term with largest degree is 1. A polynomial is *irreducible* if it cannot be written as the product of two polynomials of smaller degree. Your task is to count the number of monic, irreducible polynomials of $\mathbb{F}_p[x]$ of a given degree d.

Too difficult? Do not despair! The problem is not so hard, once you know that, in $\mathbb{F}_p[x]$, *every* monic polynomial can be written in a unique way as a factor of monic, irreducible polynomials. For instance, in $\mathbb{F}_2[x]$ there are 4 monic polynomials of degree 2 (in $\mathbb{F}_2[x]$, all polynomials are monic), but only one of them is irreducible:

$$x^2 = x \cdot x \qquad x^2+1 = (x+1)\cdot(x+1) \qquad x^2+x = x\cdot(x+1) \qquad x^2+x+1 = \text{???}$$

In $\mathbb{F}_2[x]$, there are 8 monic polynomials of degree 3, but only two of them are irreducible:

$$\begin{array}{rr}
x^3 = x\cdot x\cdot x & x^3+x^2 = x\cdot x\cdot(x+1) \\
x^3+1 = (x+1)\cdot(x^2+x+1) & x^3+x^2+1 = \text{???} \\
x^3+x = x\cdot(x+1)\cdot(x+1) & x^3+x^2+x = x\cdot(x^2+x+1) \\
x^3+x+1 = \text{???} & x^3+x^2+x+1 = (x+1)\cdot(x+1)\cdot(x+1)
\end{array}$$

Input

Input consists of several cases, each with a prime number $2 \le p \le 30$ and an integer number $2 \le d \le 30$. Additionally, we have $p^d < 10^9$.

Output

For every case, print the number of monic, irreducible polynomials in $\mathbb{F}_p[x]$ of degree d.

Sample input

```
2 2
2 3
2 4
2 30
3 2
3 3
3 4
3 19
29 6
```

Sample output

```
1
2
3
35790267
3
8
18
61171656
99133020
```

Problem information

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