Swapping parentheses P37366_en

Let P_n be the set of words with exactly n opening parentheses and n closing parentheses, such that every '`)`' matches a '`(`'. For instance,

$$P_3 = \{\text{“((()))”}, \text{“(()())”}, \text{“(())()”}, \text{“()(())”}, \text{“()()()”}\}.$$

Consider the following experiment: Choose one word w from P_n at random. Then, pick one '`(`' and one '`)`' of w, independently at random, and swap them. What is the probability that the result is also a word in P_n?

For example, let $n = 3$. If we choose $w =$ “`((()))`”, then there are exactly four swaps that produce a word in P_3, namely 2-4, 2-5, 3-4, 3-5. The rest of swaps (1-4, 1-5, 1-6, 2-6, 3-6) are incorrect. Each of the other words in P_3 has three correct swaps. Therefore, the probability for $n = 3$ is

$$\frac{1}{5}\left(\frac{4}{9}+\frac{3}{9}+\frac{3}{9}+\frac{3}{9}+\frac{3}{9}\right) = \frac{16}{45} \simeq 0.355556\ .$$

Input

Input consists of several integer numbers n between 1 and 30.

Output

For every given n, print with six digits after the decimal point the probability that swapping a random '`(`' with a random '`)`' of a random word in P_n produces a word also in P_n.

Sample input 1

```
1
2
3
10
30
```

Sample output 1

```
0.000000
0.250000
0.355556
0.585699
0.731991
```

Problem information

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