You have to program several functions. Do not use the math module.

1. Write an integer function \( \text{int_root}(n) \) that given a natural number \( n \) returns \( \lfloor \sqrt{n} \rfloor \).

2. Write a function \( \text{int_log}(a, b) \) that given natural numbers \( a \) greater than one and \( b \) greater than zero returns natural \( k \) such that \( a^k \leq b < a^{k+1} \).

3. Write a function \( \text{gcd lcm}(a, b) \) that given natural numbers \( a \) and \( b \) such that \( a \neq 0 \) or \( b \neq 0 \) returns the greatest common divisor and the least common multiple. Your code has to implement the Euclid’s algorithm.

4. Write a boolean function \( \text{is_prime}(n) \) that given a natural number \( n \) returns \text{True} if and only if \( n \) is prime.

5. In order to play table games at the casino you need some tokens. Red tokens cost 7 euros and yellow tokens cost 4. Write a function \( \text{buy_tokens}(n) \) that given a number \( n \) of euros such that \( n \geq 20 \), it returns the equivalence in tokens. When several equivalences are possible the function returns the one minimizing the total number of tokens.

6. Write a string function \( \text{max_overlap}(s, t) \) that given two strings \( s \) and \( t \) returns the longest string that is a common prefix of \( s \) and \( t \).

**Scoring**

The first function counts 15 points. Other ones count 17 point each one.

**Sample session**

```python
>>> int_root(19)
4
>>> int_log(3, 20)
2
>>> gcd_lcm(12,18)
(6, 36)
>>> is_prime(51)
False
>>> buy_tokens(50)
(6, 2)
>>> max_overlap('bugs', 'bunny')
'bu'
```